

Combining Algebraic Expressions

Combining algebraic expressions is a method for multiplying factors when unknown variables are involved. Two different ways of combining algebraic expressions will be shown here: the FOIL method and the Box method.

Combining by the FOIL Method

FOIL stands for *first*, *outer*, *inner*, and *last*. These words reference the position of the terms within the parentheses and the order they should be multiplied. The basic steps can be found in the box below.

FOIL Method Steps

1. Multiply the two First terms.
2. Multiply the two Outer terms.
3. Multiply the two Innner terms.
4. Multiply the two Last terms.
5. Combine and simplify.

Example 1:

Combine the following algebraic expressions using the FOIL Method.

$$(3x + 4)(x - 9)$$

Step 1: Multiply the two First terms.

$$(3x + 4)(x - 9)$$

$$(3x) \times (x) = 3x^2$$

Expression: $3x^2$

Step 2: Multiply the two Outer terms.

$$(3x + 4)(x - 9)$$

$$(3x) \times (-9) = -27x$$

Expression: $3x^2 - 27x$

Step 3: Multiply the two *inner* terms.

$$(3x + 4)(x - 9)$$

$$(4) \times (x) = 4x$$

$$\text{Expression: } 3x^2 - 27x + 4x$$

Step 4: Multiply the two *last* terms.

$$(3x + 4)(x - 9)$$

$$(4) \times (-9) = -36$$

$$\text{Expression: } 3x^2 - 27x + 4x - 36$$

Step 5: Simplify.

$$3x^2 - 23x - 36$$

Combining by the Box Method

The box method provides a convenient grid to keep track of all the FOIL steps.

Box Method Steps

1. Put one expression on top of a grid and the other along the left side of the grid.
2. Multiply the terms in their corresponding smaller boxes.
3. Combine and simplify.

Example 2:

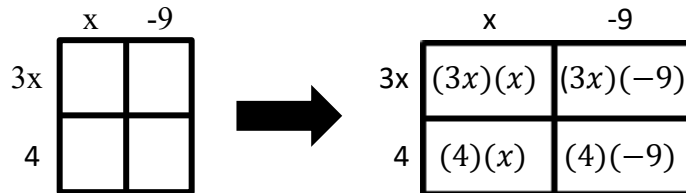
Combine the following algebraic expressions using the box method.

$$(3x + 4)(x - 9)(x - 8)$$

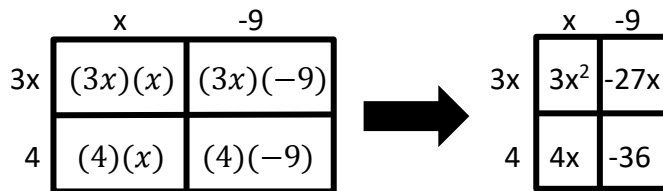
Step 1: When there are more than two expressions, start by grouping expressions into pairs.

$$(3x + 4)(x - 9)(x - 8)$$

Step 2: Draw a grid, placing one expression horizontally above the grid and the other vertically along the left side of the grid. Ensure that each smaller box has only two terms that will meet inside of it.



Step 3: Now, multiply the terms that meet in each box.



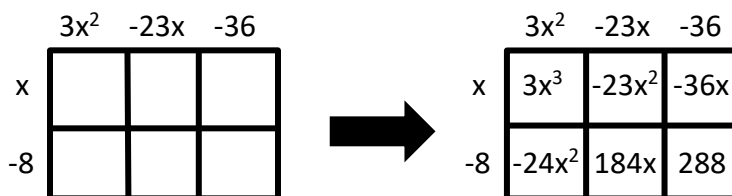
Step 4: Combine the products within the boxes. The resulting term is the combined algebraic expression for the first two expressions that were selected.

$$3x^2 - 27x + 4x - 36$$

Step 5: Simplify.

$$3x^2 - 23x - 36$$

Step 6: Repeat the process using the expression calculated from the previous step and the remaining expression from the problem.



Step 7: Combine the boxed terms again to get the final answer.

$$3x^3 - 23x^2 - 24x^2 - 36x + 184x + 288$$

Step 8: Simplify.

$$3x^3 - 47x^2 + 148x + 288$$

Sample Problems:

1. $(x - 4)(x + 3)$
2. $(x + 9)(y - 2)$
3. $(5x - 1)(2x^2 + 3)$
4. $(12b - 5)(b - 8)$
5. $(\frac{1}{2}x + 3)(x - 7)$
6. $(a + 3)(b^2 - 2b + 11)$
7. $(m - 2)(m^2 + m - 9)$
8. $(x - 4)(x + 3)(x + 11)$
9. $(n + 9)(s + 2)(n + 5)$
10. $(r - 2)(r - 10)(r + 7)(r - 3)$
11. $(x^3 - 33)(49x - 67)$
12. $(2x - 1)(4x^2 + 2x + 1)$
13. $(x^5 - x^2)(x + 3)$
14. $(x^{21} + 82)(x^2 - 4x)$
15. $(x^2 - 4x + 2)(x^2 + 8x - 7)$
16. $b(y - 5)$
17. $(x^2 - 13)(x + 1)$
18. $(mn + 2)(m^2 - 4m + 1)$
19. $(5x + 4)(5x - 4)$
20. $(x^3 - 2x^2 + 8x + 3)(2x^4 + x^2 - 11x + 101)$

Answers:

1. $x^2 - x - 12$
2. $xy - 2x + 9y - 18$
3. $10x^3 - 2x^2 + 15x - 3$
4. $12b^2 - 101b + 40$
5. $\frac{1}{2}x^2 - \frac{1}{2}x - 21$
6. $b^2a + 3b^2 - 2ba - 6b + 11a + 33$
7. $m^3 - m^2 - 11m + 18$
8. $x^3 + 10x^2 - 23x - 132$
9. $sn^2 + 2n^2 + 14sn + 28n + 45s + 90$
10. $r^4 - 8r^3 - 49r^2 + 332r - 420$
11. $49x^4 - 67x^3 - 1617x + 2211$
12. $8x^3 - 1$
13. $x^6 + 3x^5 - x^3 - 3x^2$
14. $x^{23} - 4x^{22} + 82x^2 - 328x$
15. $x^4 + 4x^3 - 37x^2 + 44x - 14$
16. $by - 5b$
17. $x^3 + x^2 - 13x - 13$
18. $m^3n - 4m^2n + 2m^2 + mn - 8m + 2$
19. $25x^2 - 16$
20. $2x^7 - 4x^6 + 17x^5 - 7x^4 + 131x^3 - 287x^2 + 775x + 303$