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Simplifying Circuits

A circuit is any closed loop between two or more points through which electrons may flow from a voltage or current source. Circuits range in complexity from one, basic component to a variety of components arranged in different ways. This handout will discuss the basics of circuits and the associated laws required to analyze and simplify them.

You can navigate to specific sections of this handout by clicking the links below.

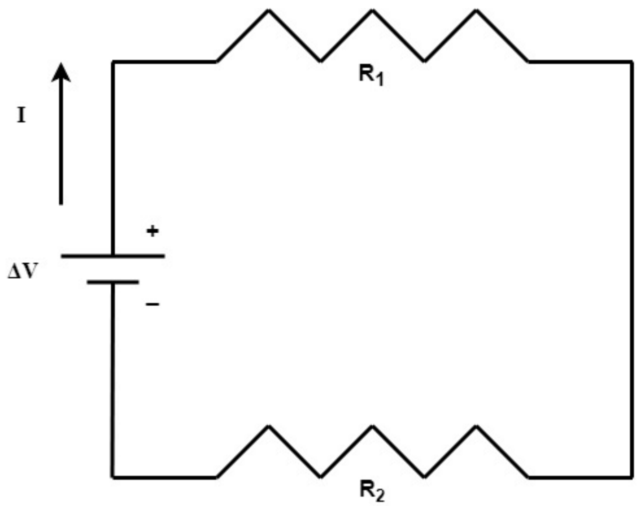
Key Terms

The following table defines key terms needed to work with circuits.

Basic Terms	**Definition**	**SI Units**	**Formula**
Resistance “R”	The ratio of voltage (V) across a conductor to the current (I) in the conductor.	Ohms (Ω)	R = V/I
Current “I”	The amount of charge passing through a particular region over a set amount of time.	Amperes (A)	I = V/R
Voltage “V”	A measure of potential difference/electric potential across a circuit.	Volts (V) = $(\frac{1\ \text{Coulomb}}{\text{Second}})$	V = I*R
Power “P”	The rate at which electric energy travels through a circuit to a given point.	Watt (W) = $(\frac{1\ \text{Joule}}{\text{Second}})$	P = I*V

Series and Parallel

There are two basic configurations of resistors within circuits: series and parallel. In a series configuration, the resistors are connected in a single path so that the charge must travel through them in sequence.



For circuits containing resistors in a series configuration, the same amount of current will flow through every component, but the voltage will change. The equivalent resistance (represented as R_E or R_T if there is only one resistance remaining) is calculated by applying the following equation:

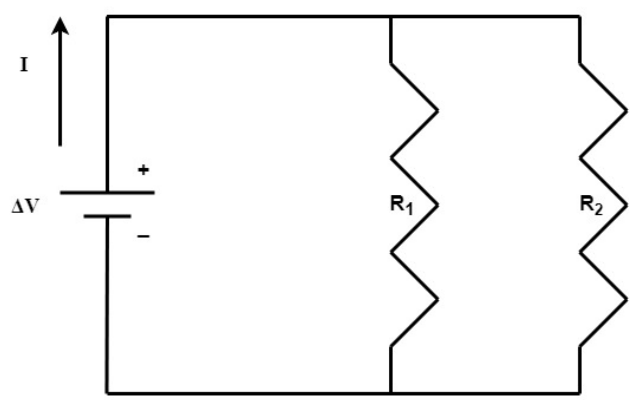
$$R_T = R_1 + R_2 + \cdots + R_N$$

Resistors in Series

A parallel configuration of resistors, however, allows multiple paths for the charge to travel throughout the circuit.

The resistors in the circuit shown on the right are in a parallel configuration, and the voltage will remain the same across each resistor. The current will change. The equivalent resistance is calculated using the following formula:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$



Resistors in Parallel

Simplifying Circuits

In reality, most circuits are not in a basic series or parallel configuration, but rather consist of a complex combination of series and parallel resistances. The key to simplifying circuits is to combine complex arrangements of resistors into one main resistor. The general rules for solving these types of problems are as follows:

1. Start simplifying the circuit as far away from the voltage source as possible.
    a. Analyze the circuit to find a section in which all resistors are either series or parallel.
2. Reduce series and parallel configurations into equivalent resistances $(\mathrm{R_E})$.
    a. Moving closer to the voltage source, continue combining resistors until one, total resistance $(\mathrm{R_T})$ remains.
3. Reconstruct the circuit step-by-step while analyzing individual resistors.
    a. Find Voltage (V) and Current (I).

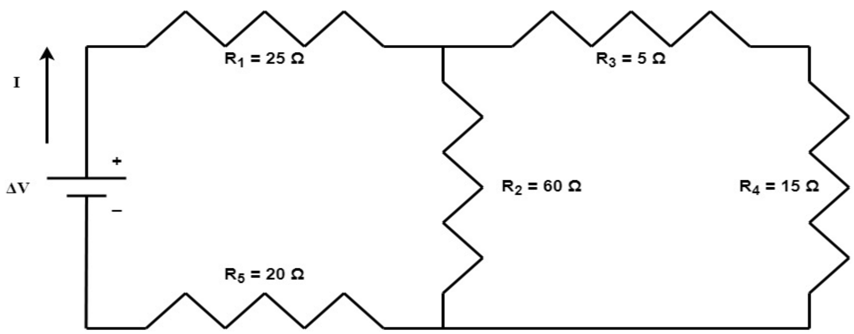
A useful strategy when analyzing circuits is to keep track of all the calculated properties within a circuit with a chart that contains the values for the resistances, currents, and voltages for each resistor within the circuit. The chart will be set up as follows:

Component	Resistance (Ω)	Current (mA)	Voltage (V)
R_1			
R_2			
R_3			
R_4			
R_5			

Example

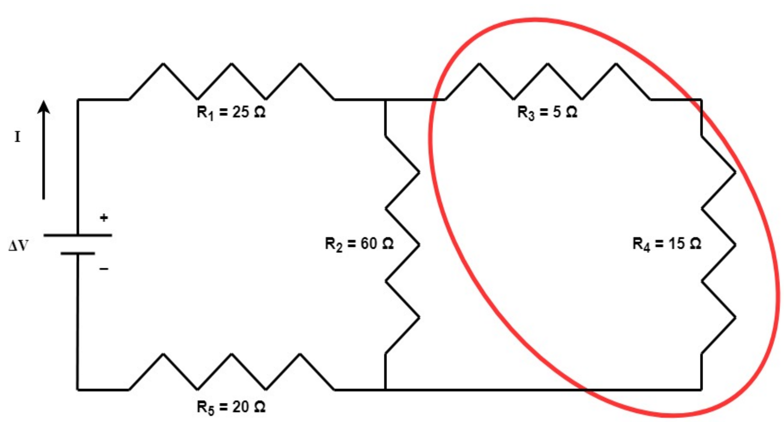
Find the current and voltage across each resistor of the following circuit, if ΔV = 18 V.

At first glance, this circuit falls under neither of the two configurations discussed earlier—series nor parallel—rather it contains a combination of the two. In order to find the current and voltage across each resistor, simplify the circuit to a basic state (containing only a single resistor). Then, reconstruct it step-by-step. Following the aforementioned rules, the first step is to analyze the circuit. To do this, find a section where all resistors are in either series or parallel and that is furthest from the voltage source.



Step 1 – Where to Start

By looking at the circuit shown below, resistors R_3 and R_4 are the best fit for the previously stated rule regarding where to begin analyzing. Since these two resistors are in a series configuration, combine them as follows and calculate their equivalent resistance using the series equation. Recall the equation for resistance in a series configuration from earlier:



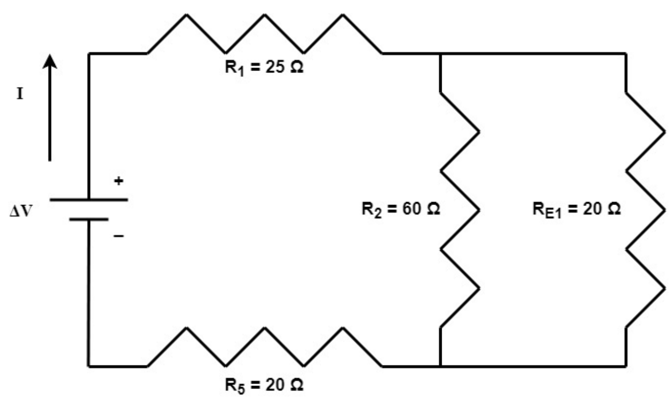
$$R_T = R_1 + R_2 + \cdots + R_N$$

$$\mathbf{R_{E1} = 5 + 15 = 20\ \Omega}$$

When simplifying into equivalent resistances, it is necessary to add a new row in the chart for each R_E created within the circuit. For example, since R_{E1} was just calculated, there should be a new row added to the bottom of the chart as follows:

Component	Resistance (Ω)	Current (mA)	Voltage (V)
R_1	25		
R_2	60		
R_3	5		
R_4	15		
R_5	20		
R_{E1}	20		

Step 2a - Simplify



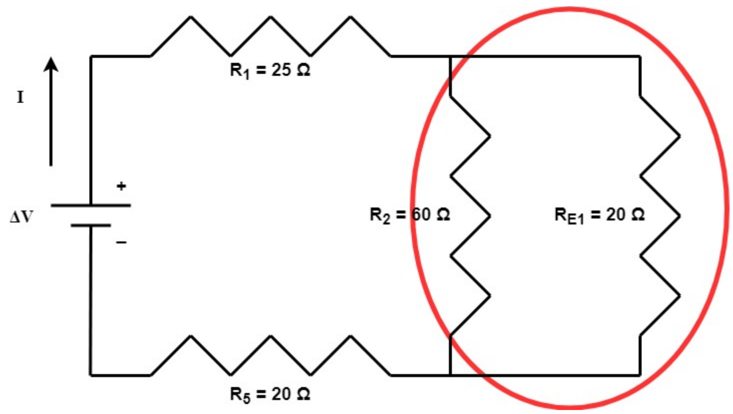
Recall the equation for resistance in a parallel configuration from earlier:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$$\mathbf{\frac{1}{R_{E2}} = \frac{1}{60} + \frac{1}{20} = \frac{1}{15}}$$

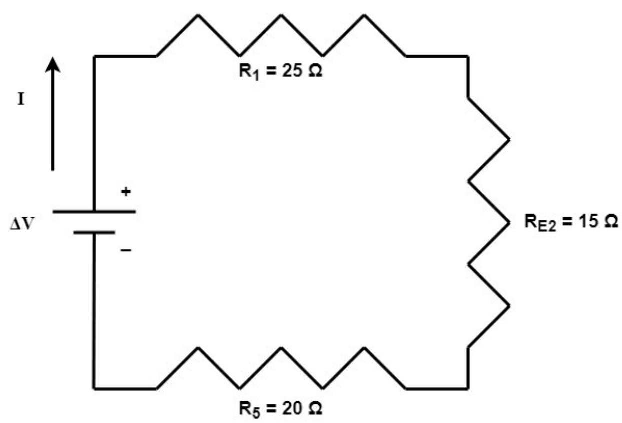
By simplifying the resistors in series, R_3 and R_4 become one equivalent resistance, labeled R_{E1} with a value of 20 Ohms. Now, repeat the process, but this time using resistors R_2 and the newly created R_{E1}.

Step 2b – Continue Simplifying Remaining Resistors



This time the equation for a parallel configuration must be used to find R_2 and R_{E1}'s equivalent resistance since they are in a parallel configuration.

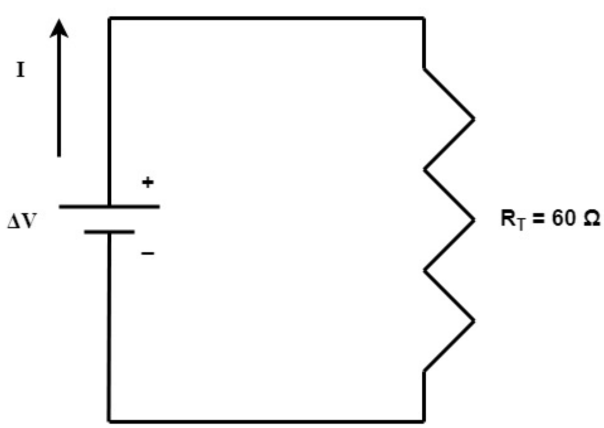
Step 2c



Now the circuit has been simplified to three resistors, which are all in a series configuration. Combine these resistors using the series addition equation:

$$\mathbf{R_1 + R_{E2} + R_5 = R_{E3}}$$

Step 2d



This final combination leaves the circuit with one resistor, which will be titled "R_T" because it is the total resistance of the circuit. The system's total resistance is 60 Ohms. Therefore, the formulas from the chart on the first page may now be applied to begin finding the properties of the original resistors.

Because there is only one resistor in the circuit, the voltage flowing though the resistor must be equivalent to the amount coming through the voltage source (18V). With the resistance and voltage known, there is only one unknown value in The Ohm's Law equation (V = I*R), so the current (I) may now be calculated:

R_T: R = 60 Ω

V = 18.0 V

$I = \frac{V}{R} = \frac{18}{60}$ = .3 A = 300 mA

Current is often calculated to be a decimal when solving circuits, so it is common practice to write the value in terms of milliamps (mA).

Now voltage (V), current (I), and resistance (R) are known for R_T (or R_{E3}), and the circuit can be rebuilt. The Ohm's Law equation will be used during this process to evaluate the other components within the circuit. At this point, the chart should have all resistance values filled in along with the voltage and current for R_T as follows:

Component	Resistance (Ω)	Current (mA)	Voltage (V)
R_1	25		
R_2	60		
R_3	5		
R_4	15		
R_5	20		
R_{E1}	20		
R_{E2}	15		
$R_{E3} = R_T$	60	300	18.0

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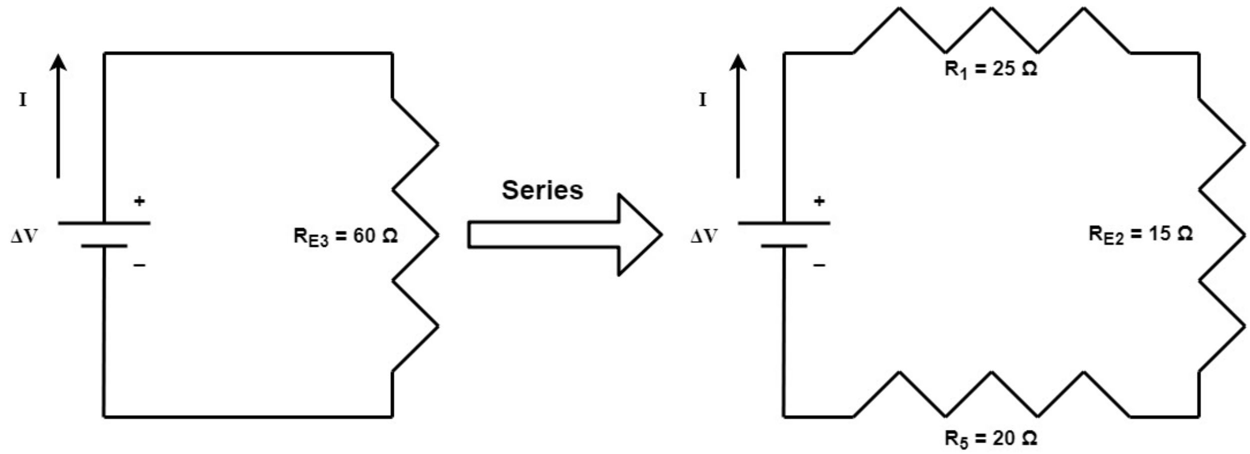
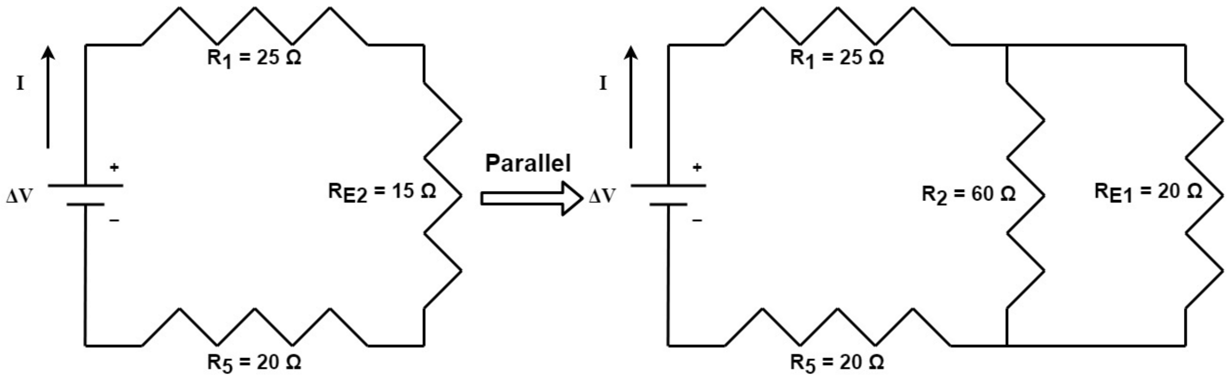
Step 3 – Reconstruct & Solve



To solve for the current and voltage across all of the resistors, undo the most recent change made when simplifying the circuit, in this case steps 2b and 2c. In the process of undoing a step, first determine whether the resistors are in parallel or series configuration. This will determine which value from the simplified resistor will remain constant and carry over, in this case, $R_T = R_1 + R_{E2} + R_5$. Because these three resistors are in a series setup, their current equals the current flowing through R_T, which is 300mA. Using V=I*R, the voltage for each resistor can be solved using their current (300mA) and their resistance given at the beginning of the problem.

R_1:	R = 25 Ω I = .3 A = 300 mA V = (.3)*(25) = 7.5 V	R_{E2}:	R = 15 Ω I = .3 A = 300 mA V = (.3)*(15) = 4.5 V	R_5:	R = 20 Ω I = .3 A = 300 mA V = (.3)*(20) = 6 V

Step 4a



Continue to rebuild the circuit until each resistor's voltage and current has been found. R_{E2} was comprised of two resistors in parallel configurations, R_2 and R_1. As stated earlier, for parallel configurations the voltage remains constant across all the resistors. Therefore, R_2 and R_{E1} will have the same voltage across them as R_{E2}. Now, use V = I*R to calculate the current through both R_2 and R_{E1}.

R_2: R = 60 Ω

I = $\frac{4.5}{60}$ = .075 A = 75 mA

V = 4.5 V

R_{E1}: R = 20 Ω

I = $\frac{4.5}{20}$ = .225 A = 225 mA

V = 4.5 V

Step 4b

Next, R_3 and R_4 were combined in series configuration to create R_{E1} in Step 1. Follow Step 4a to find the voltage running across each resistor.

R_3: R = 5 Ω

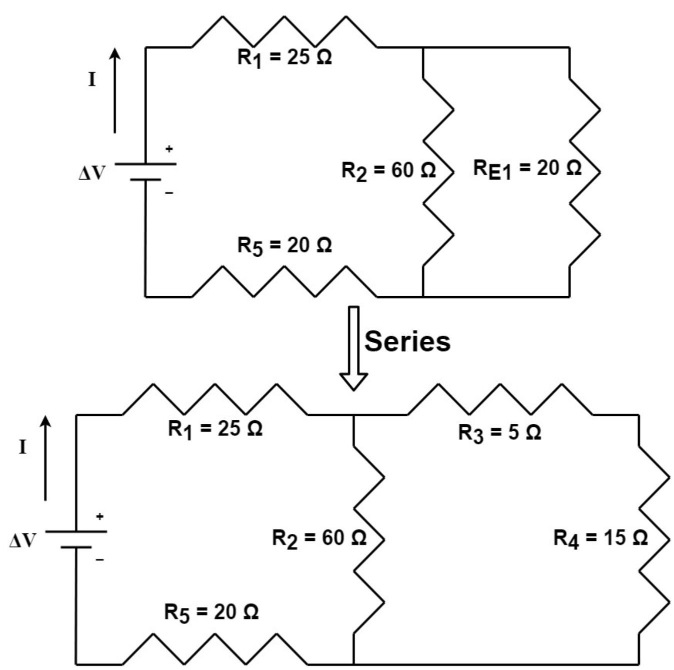
I = .225 A = 225 mA

V = (5)*(.225) = 1.125 V

R_4: R = 15 Ω

I = .225 A = 225 mA

V = (15)*(.225) = 3.375 V

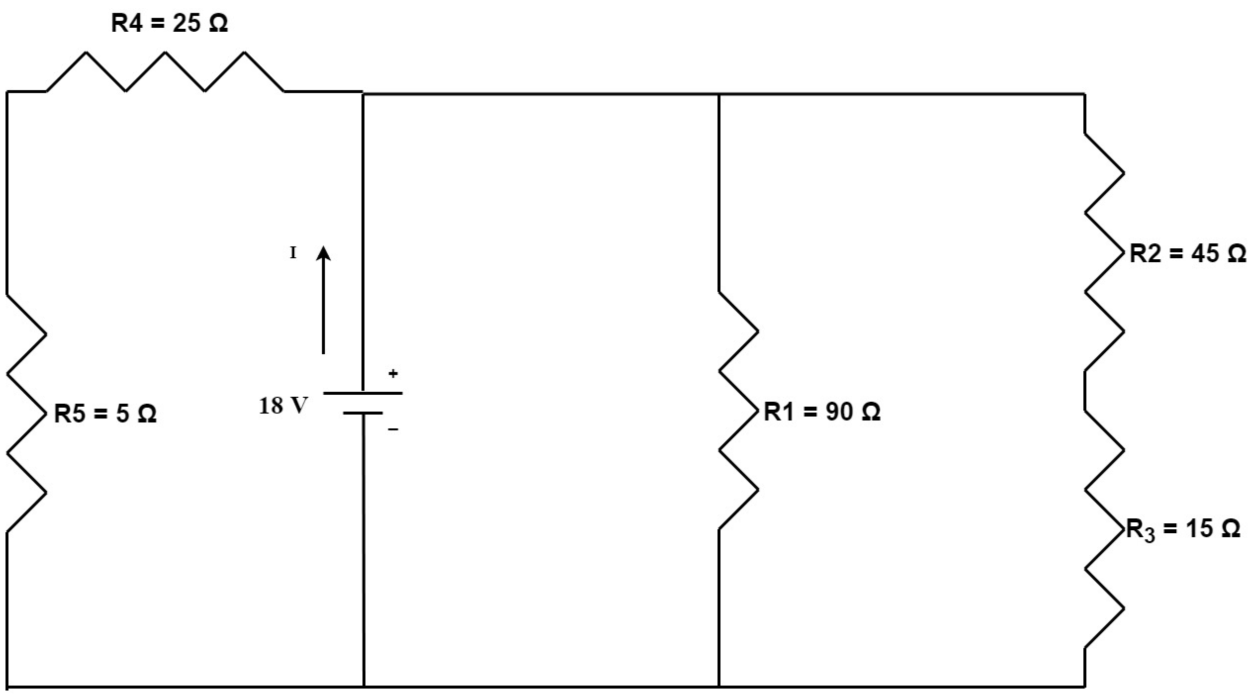


Component	Resistance (Ω)	Current (mA)	Voltage (V)
R_1	25	300	7.5
R_2	60	75	4.5
R_3	5	225	1.125
R_4	15	225	3.375
R_5	20	300	6
R_{E1}	20	225	4.5
R_{E2}	15	300	4.5
$R_{E3} = R_T$	60	300	18.0

As the completed chart above shows, the voltage, current, and resistance of each resistor within the system are now known. Using this method of simplifying circuits is helpful in determining the properties of individual resistors within a complex circuit. For more practice with this method, see the following pages containing example problems.

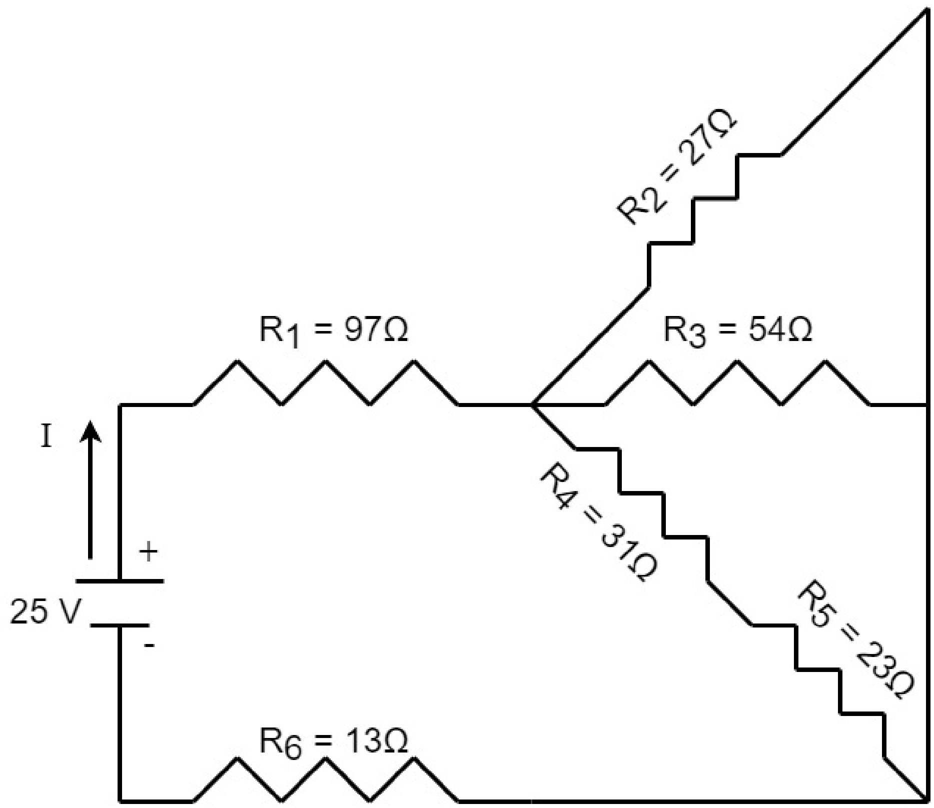
Practice Problems: Simplifying Circuits

<u>Problem 1:</u>



Find the voltage and current (in mA) across resistors 1-5 as well as the total resistance.

Problem 2:



Find the voltage and current (in mA) across resistors 1-5 as well as the total resistance.

Solutions:

1)

Component	Resistance (Ω)	Current (mA)	Voltage (V)
R_1	90	200	18
R_2	45	300	13.5
R_3	15	300	4.5
R_4	25	600	15
R_5	5	600	3
R_T	16.36	1100	18

2)

Component	Resistance (Ω)	Current (mA)	Voltage (V)
R_1	97	202.4	19.63
R_2	27	101.1	2.73
R_3	54	50.6	2.73
R_4	31	50.6	1.57
R_5	23	50.6	1.16
R_6	13	202.4	2.63
R_T	123.5	202.4	25

Equivalent Resistances have been omitted due to the existence of multiple possible answers