How to Graph Trigonometric Functions

This handout includes instructions for graphing processes of basic, amplitude shifts, horizontal shifts, and vertical shifts of trigonometric functions.

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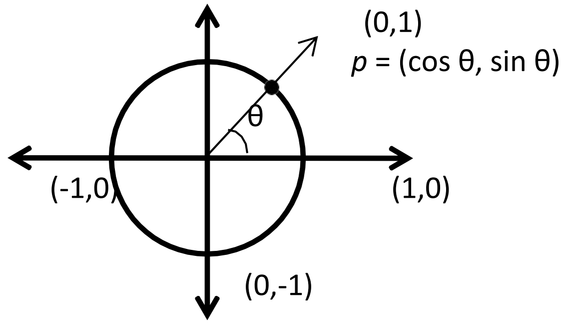
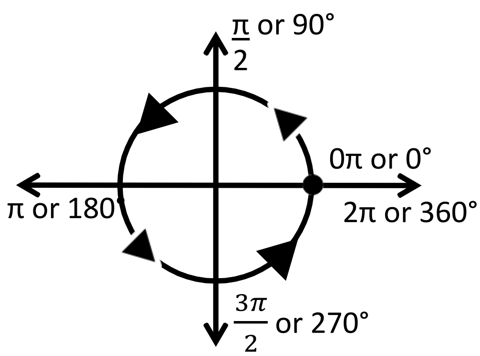
The Unit Circle and the Values of Sine and Cosine Functions

The unit circle is a circle with a radius that equals 1. The angle θ is formed from the ϕ (phi) ray extending from the origin through a point *p* on the unit circle and the *x*-axis; see diagram below. The value of $\sin \theta$ equals the y-coordinate of the point *p* and the value of $\cos \theta$ equals the x- coordinate of the point *p* as shown in the diagram below.



This unit circle below shows the measurements of angles in radians and degrees. Beginning at 0π, follow the circle counter-clockwise. As angle θ increases to $\frac{\pi}{2}$ radians or 90°, the value of cosine (the *x*-coordinate) decreases because the point is approaching the y-axis. Meanwhile, the value of sine (the y-coordinate) increases. When one counter-clockwise revolution has been completed, the point has moved 360° or 2π.



Graphing Sine and Cosine Functions y = sin x and y = cos x

There are two ways to prepare for graphing the basic sine and cosine functions in the form **y = sin x** and **y = cos x**: evaluating the function and using the unit circle.

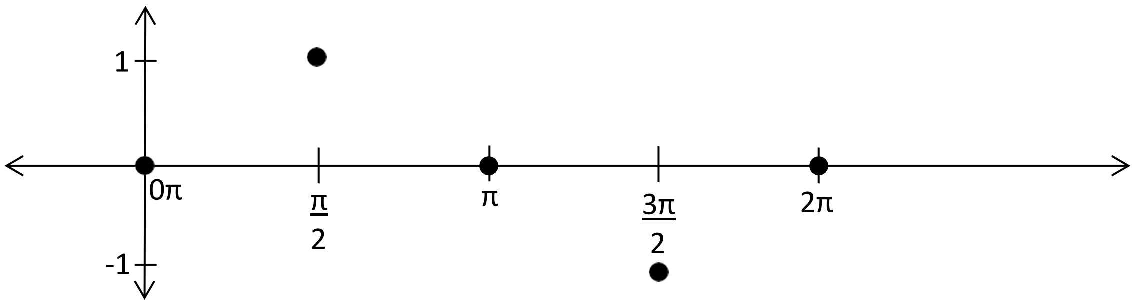
To evaluate the basic sine function, set up a table of values using the intervals 0π, $\frac{\pi}{2}$, $\frac{3\pi}{2}$, and 2π for *x* and calculating the corresponding y value.

f(x) or y = sin x

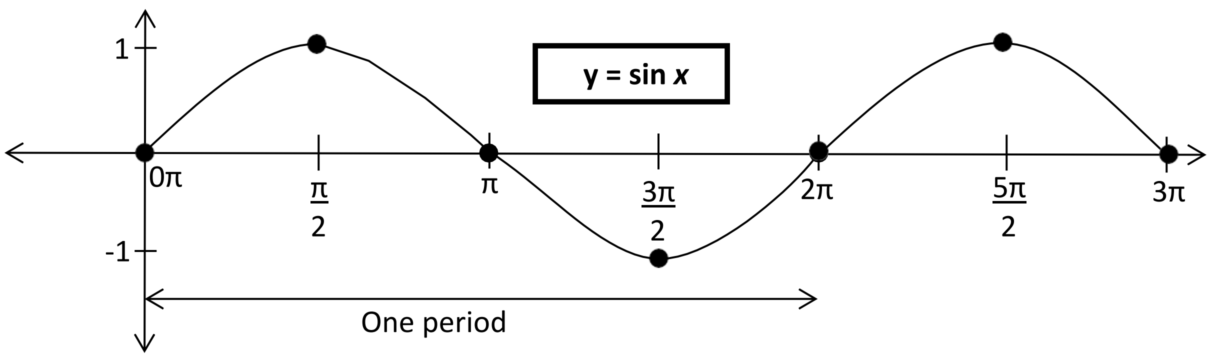
f(x) or y	x
0	0π
1	$\frac{\pi}{2}$
0	π
-1	$\frac{3\pi}{2}$
0	2π

To use the unit circle, the x-coordinates remain the same as within the list above. To find the y- coordinate of the point to graph, first locate the point *p* on the unit circle that corresponds to the angle θ given by the x-coordinate. Then, use the y-coordinate of the point *p* as the y value of the point to graph.

To draw the graph of one period of sine or **y = sin x**, label the x-axis with the values 0π, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π. Then plot points for the value of **f(x)** or **y** from either the table or the unit circle.



Other points may be added for the intermediate values between those listed above to obtain a more complete graph, and a best fit line can be drawn by connecting the points. The figure below is the completed graph showing one and a half periods of the sine function.



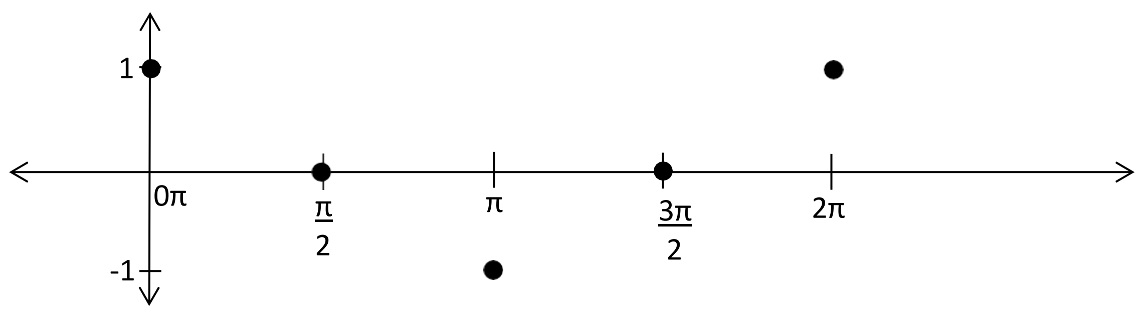
The graph of the cosine function **y = cos x** is drawn in a similar manner as the sine function. Using a table of values:

f(x) or y = cos x

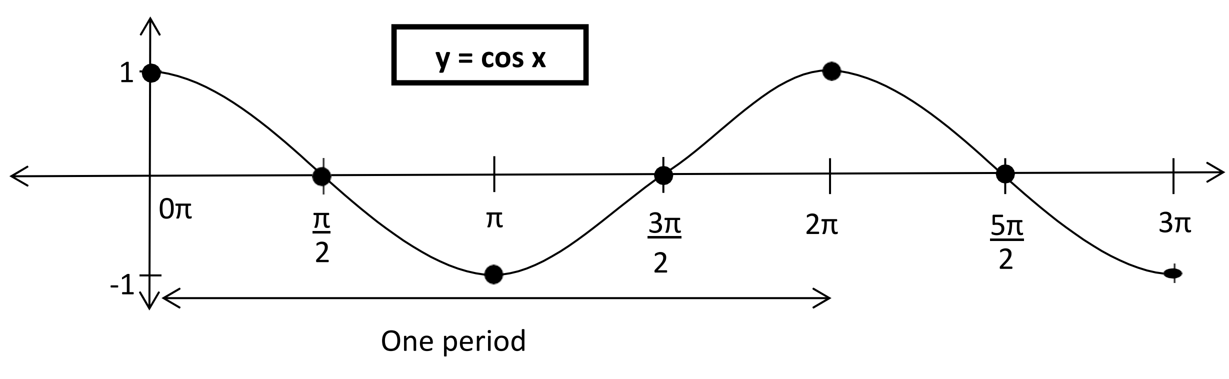
f(x) or y	x
1	0π
0	$\frac{\pi}{2}$
-1	π
0	$\frac{3\pi}{2}$
1	2π

To use the unit circle, the x-coordinate remains the same as the list on the previous page. To find the y-coordinate of the point to graph, first locate the point *p* on the unit circle that corresponds to the angle θ given by the *x*-coordinate. Then, use the x-coordinate of the point *p* as the *y* value of the point to graph.

To draw the graph of one period of cosine or **y = cos x**, label the x-axis with the values 0π, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π. Then plot points for the value of f(x) or y from either the table or the unit circle.

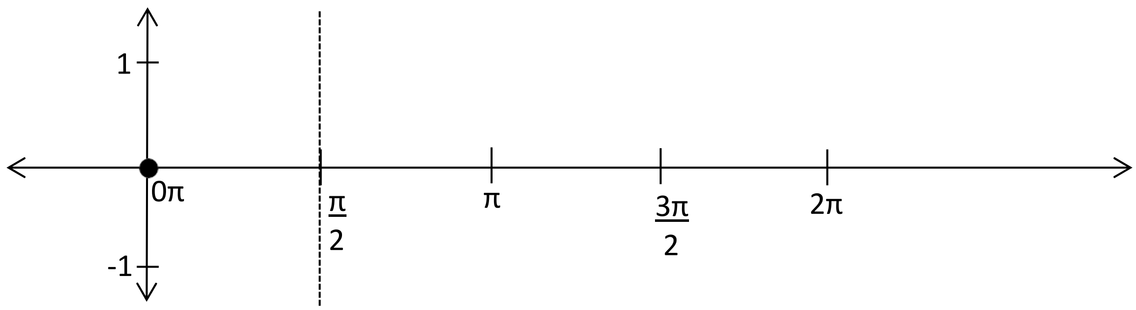


Add other points as required for the intermediate values between those above to obtain a more complete graph, and draw a best fit line connecting the points. The graph below shows one and a half periods.

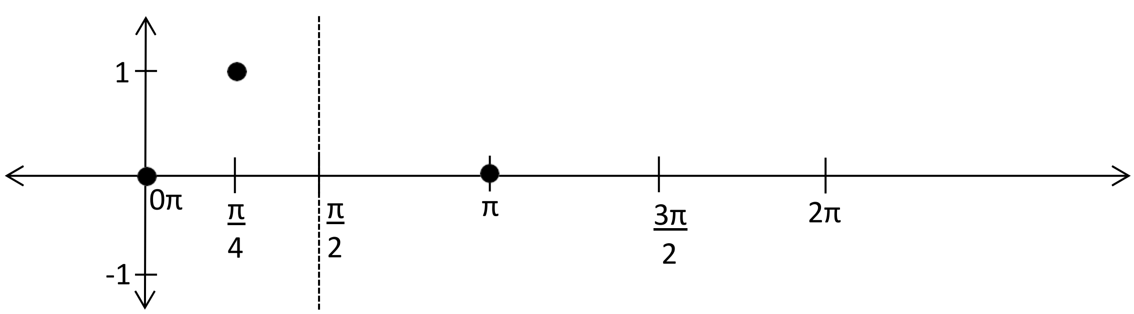


Graphing the Tangent Function y = tan x

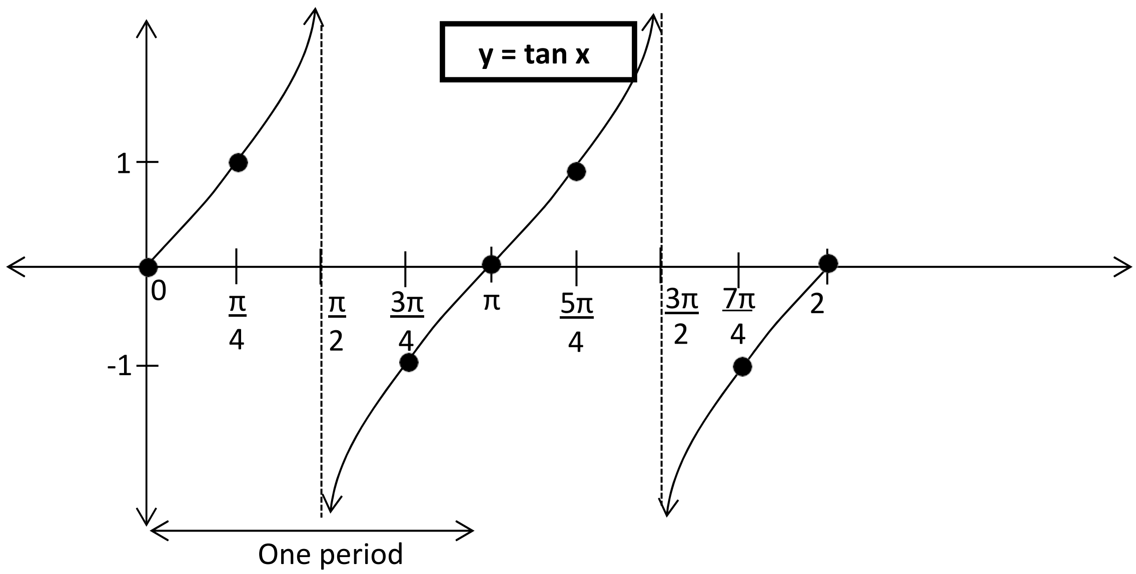
The tangent value at angle θ is equal to the sine value divided by the cosine value $\left(\frac{Sine\ Value}{Cosine\ Value}\right)$ of the same angle θ. The value of tangent at 0π for the unit circle is $\frac{0}{1}$, which is equivalent to 0. The value of tangent at $\frac{\pi}{2}$ is $\frac{1}{0}$. This yields a divide by 0 error or undefined (try this in your calculator). Therefore, the tangent function is undefined at $\frac{\pi}{2}$. This is illustrated by drawing an asymptote (vertical dashed line) at $\frac{\pi}{2}$. See the figure below.



The value of tangent at π is $\frac{0}{1}$, which results in 0. To determine how the tangent behaves between 0π and the asymptote, find the sine and cosine values of $\frac{\pi}{4}$, which is half way between 0π and $\frac{\pi}{2}$. Looking at the handout *Common Trigonometric Angle Measurements*, the tangent of $\frac{\pi}{4}$ is $\frac{\sqrt{2}}{2}$ (sine) divided by $\frac{\sqrt{2}}{2}$ (cosine). Flipping the cosine value and multiplying gives $\frac{\sqrt{2}}{2} \times \frac{2}{\sqrt{2}}$ which simplifies to 1. The value of tangent at $\frac{\pi}{4}$ is therefore 1. These points have been added to the graph below.



Next, calculate the value of tangent for $\frac{3\pi}{4}$. Consulting with *Common Trigonometric Angle Measurements*, the tangent of $\frac{3\pi}{4}$ is $\frac{\sqrt{2}}{2}$ (sine) divided by $-\frac{\sqrt{2}}{2}$ (cosine). This simplifies to a tangent value of -1. Now, draw the tangent function graph so that the line approaches the asymptote without touching or crossing it. The image on the next page shows the completed graph of one and a half periods of the tangent function.



The period of the basic tangent function is π, and the graph will repeat from π to 2π.

The Form y = A sin(Bx + C) + D

The form **y = A sin(Bx + C)** is the general form of the sine function. From this general form of the **sine** function, the amplitude, horizontal, phase, and vertical shifts from the basic trigonometric forms can be determined.

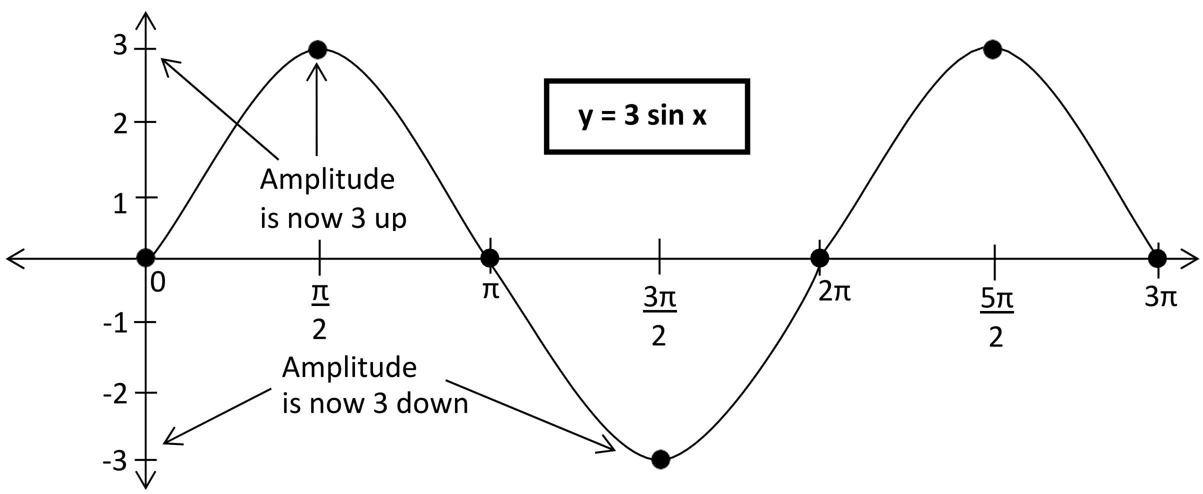
> ***A*** : modifies the amplitude in the ***y*** direction above and below the center line
>
> ***B*** : influences the period and phase shift of the graph
>
> ***C*** : influences the phase shift of the graph
>
> ***D*** : shifts the center line of the graph on the y-axis

Amplitude Shifts of Trigonometric Functions

The basic graphs illustrate the trigonometric functions when the A value is 1. This $A = 1$ is used as an amplitude value of 1. If the value A is not 1, then the absolute value of A value is the new amplitude of the function. Any number $|A|$ greater than 1 will vertically stretch the graph (increase the amplitude) while a number $|A|$ smaller than 1 will compress the graph closer to the *x*-axis.

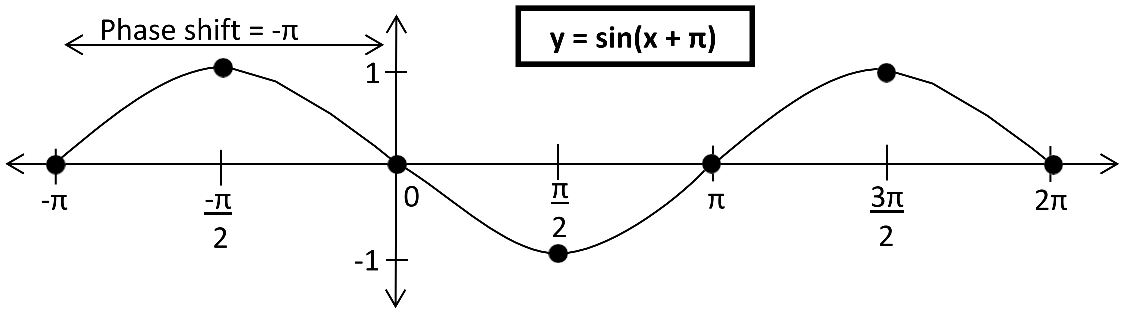
Example: Graph y = 3 sin x.

Solution: The graph of **y = 3 sin x** is the same as the graph of **y = sin x** except the minimum and maximum of the graph has been increased to -3 and 3 respectively from -1 and 1.



Horizontal Shifts of Trigonometric Functions

A horizontal shift is when the entire graph shifts left or right along the *x*-axis. This is shown symbolically as **y = sin(Bx – C)**. Note the minus sign in the formula. To find the **phase shift** (or the amount the graph shifted) divide **C** by **B** $\left(\frac{C}{B}\right)$**.** For instance, the phase shift of **y = cos(2x – π)** can be found by dividing **π (C)** by **2 (B)**, and the answer is $\frac{\pi}{2}$**.** Another example is the phase shift of **y = sin(-2x – π)** which is **–π (C)** divided by **-2 (B)**, and the result is $\frac{\pi}{2}$. Be careful when dealing with the signs. A positive sign takes the place of the double negative signs in the form **y = sin(x + π)**. The ***C*** is negative because this example is also written as **y = sin(x-(- π))**, which produces the negative π phase shift (graphed below). It is important to remember a positive phase shift means the graph is shifted right or in the positive direction. A negative phase shift means the graph shifts to the left or in the negative direction.



Period Compression or Expansion of Trigonometric Functions

The value of ***B*** also influences the period, or length of one cycle, of trigonometric functions. The period of the basic sine and cosine functions is 2π while the period of the basic tangent function is π. The period equation for sine and cosine is: **Period =** $\frac{2\pi}{|B|}$. For tangent, the period equation is: **Period =** $\frac{\pi}{|B|}$. Period compression occurs if the absolute value of ***B*** is greater than **1**; this means the function oscillates more frequently. Period expansion occurs if the absolute value of ***B*** is less than **1**; this means the function oscillates more slowly.

The starting point of the graph is determined by the phase shift. To determine the key points for the new period, divide the period into **4** equal parts and add this part to successive x values beginning with the starting point.

Example: Graph **y = sin(2x – π)**

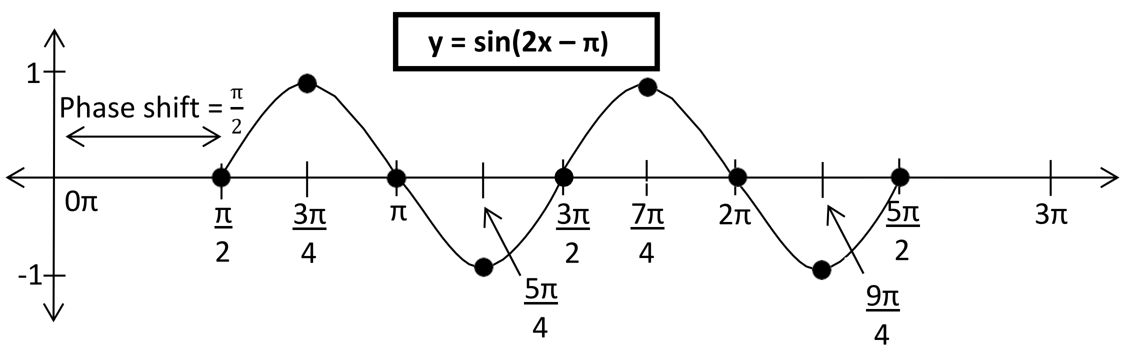
Solution: ***A*** **= 1,** ***B*** **= 2,** ***C*** **= π,** ***D*** **=** not written so **0**

The amplitude of ***A*** is 1

The horizontal (phase) shift is $\frac{C}{B} = \frac{\pi}{2} = \frac{\pi}{2}$

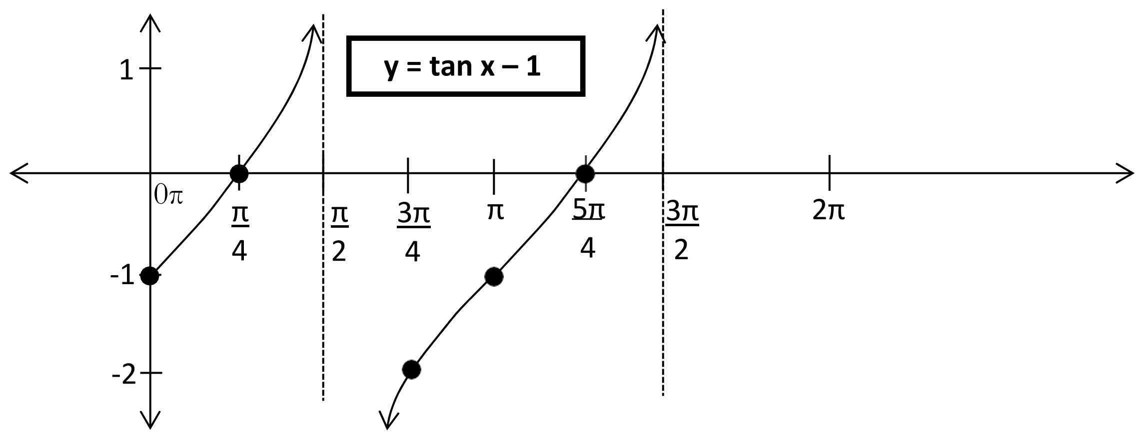
The period changes to $\frac{2\pi}{|B|} = \frac{2\pi}{|2|} = \pi$

The starting point is $\frac{\pi}{2}$. To find the 5 key points, divide π by 4 to obtain the value of the 4 equal parts: $\frac{\pi}{4}$. Then add $\frac{\pi}{4}$ to the starting point and each successive value of x to find the remaining four points. The 5 key points are: $\frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}$. Once the key points are determined, evaluate the equation at these values and plot the resulting points.



Vertical Shifts of Trigonometric Functions

A vertical shift occurs when the entire graph shifts up or down along the y-axis. This is shown symbolically as y = sin (x) + D. This is different from horizontal shifts because there are no parentheses around the D, and D is always a constant. In these examples, the graph of **y = sin (x) + 2** would shift the centerline, thus the entire graph, up 2 units, and **y = tan (x) – 1** would shift the entire graph down 1 unit.



Strategies, Summary, and Exercises

By using the following guidelines, it will make trigonometric functions easier to graph:

1) Recall the values of sine and cosine on the unit circle.
2) Identify the **amplitude shift** value **A**, in y = **A** sin x, y = **A** cos x, etc.
3) Identify the **horizontal shift** value $\frac{C}{B}$, in y = sin(**B**x – **C**), y = cos(**B**x – **C**), etc.
4) The **period compression or expansion** of the graph is determined by dividing the period of the basic function by the absolute value of **B**.
5) Identify the **vertical shift** value **D**, in y = sin (x) + **D**, y = cos (x) + **D**, etc.
6) The phase shift (if any) is the starting point to graphing the function. Divide the period by 4 to determine the 5 points to graph on the x-axis.
7) Use the 5 points to determine corresponding y-coordinate.

Exercises:

Find the amplitude shift, horizontal shift, period, and vertical shift of the following and graph one period.

1) $y = \frac{1}{4}\sin x + 1$

2) $y = 2 \sin x$

3) $y = \frac{1}{2}\cos(-x + \pi) - 1$

Solutions to Exercises

1) y = $\frac{1}{4}$sin x + 1

First find the values of A, B, C, and D

A = $\frac{1}{4}$, B = 1, C = not written which means 0, and D = 1

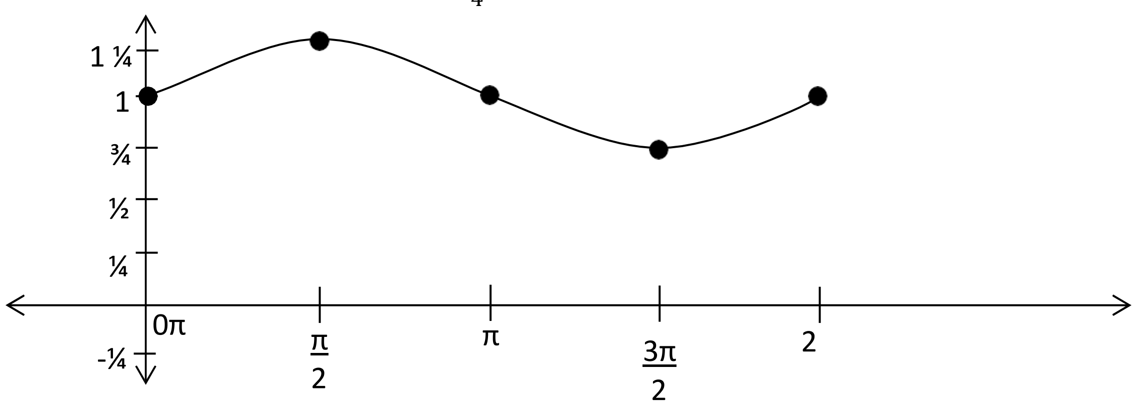
Amplitude shift is A, A = $\frac{1}{4}$

Horizontal shift is $\frac{C}{B} = \frac{0}{1} = 0$

Period is $\frac{2\pi}{|1|}$ = 2π

Vertical shift is D = 1

Graph of one period of y = $\frac{1}{4}$ sin x + 1



2) y = 2 sin x

First find the values of A, B, C, and D

A =2, B = 1, C = not written which means 0, and D = 0

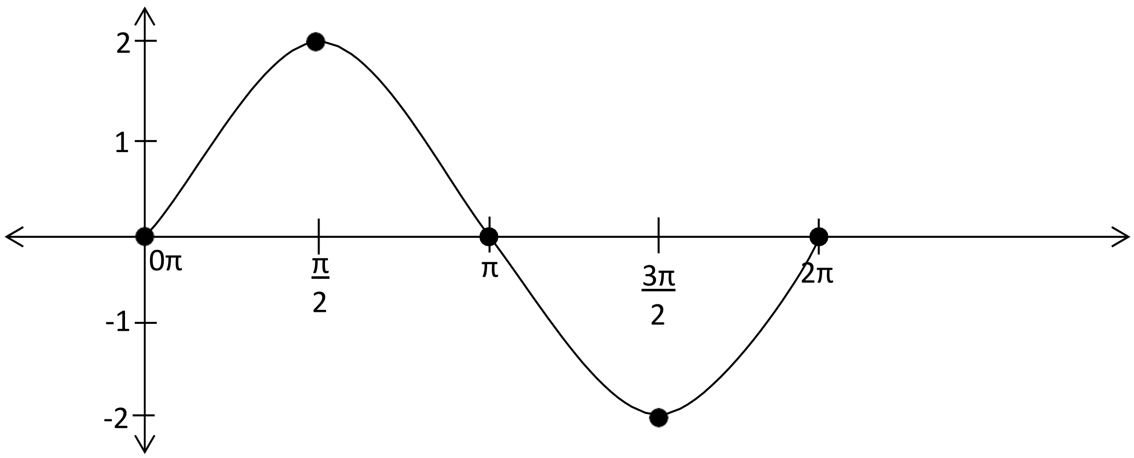
Amplitude shift is A, A = 2

Horizontal shift is $\frac{C}{B} = \frac{\mathbf{0}}{\mathbf{1}} = 0$

Period is $\frac{\mathbf{2\pi}}{|\mathbf{1}|}$ = 2π

Vertical shift is D = 0

Graph of one period of 2sin x



3) $y = \frac{1}{2}\cos(-x + \pi) - 1$

First find the values of A, B, C, and D

$A = \frac{1}{2}$, $B = -1$, $C = -\pi$, and $D = -1$

Amplitude shift is A, $A = \frac{1}{2}$

Horizontal shift is $\frac{C}{B} = \frac{-\pi}{-1} = \pi$

Period is $\frac{2\pi}{|-1|} = 2\pi$

Vertical shift is $D = -1$

Graph of one period of $y = \frac{1}{2}\cos(-x + \pi) - 1$